

Thin Disk Models of Anomalous X-ray Pulsars

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We discuss the options of the fall-back disk model of Anomalous X-Ray Pulsars (AXPs). We argue that the power-law index of the mass inflow rate during the propeller stage can be lower than those employed in earlier models. We take into account the effect of the super-critical mass inflow at the earliest stages on the inner radius of the disk and argue that the system starts as a propeller. Our results show that, assuming a fraction of the mass inflow is accreted onto the neutron star, the fall-back disk scenario can produce AXPs for acceptable parameters.

1. Introduction

The models for explaining the X-ray energy output of Anomalous X-ray Pulsars (AXPs, see [1] for a review) can be classified in two groups. Magnetar models [2,3,4] assign the persistent emission and bursts [5,6] to the extra-ordinary magnetic field of the neutron star while accretion models [7,8,9,10,11,12] employ neutron stars with intrinsic properties similar to other neutron stars and try to explain the X-ray emission and spin properties by the way matter is supplied. AXPs are very similar to Soft Gamma Ray Repeaters (SGRs) [13,14,15] and probably have the same nature.

Magnetar model is successful in modelling the SGR/AXP bursts and spin-down rates but can not explain the period clustering [16] except in one set of field decay models under special conditions [17]. Interaction of the neutron star with a fall-back disk towards equilibrium can naturally account for the period clustering.

Mereghetti & Stella (1995)[7] proposed that AXPs were accreting from a very low mass companion. There are severe upper limits for the mass of such a companion and no orbital motion signatures have been detected. Van Paradijs et al. (1995) [8] proposed a fossil disk model in which the disk results from the final spiral-in destruction of the binary. Such a model addresses the absence of the binary companion but can not ac-

count for the young ages of AXPs indicated by the supernova remnant (SNR) associations [18,19]. Fall-back disk models [9,10,11,12] can address the young ages of AXPs and the absence of a mass donor companion. In all fall-back disk models the spin down of the neutron star is achieved by the propeller effect [20,21,22,23,24,25,26] in which the rapid rotation of the neutron star prevents inflowing matter from accreting onto the surface of the star. Even when the star is slow enough that accretion can commence, it can continue to spin-down until torque equilibrium. AXPs must be those spinning-down accreting systems near torque equilibrium.

Recent detections in the infrared (and optical) support the existence of a disk [27]. The magnetar models do not have a prediction for infrared emission. Fall-back disks are not necessarily geometrically thin but, as the only models available for calculation, they warrant a careful study to decide whether interaction with a fall-back disk can produce neutron stars with AXP properties at the ages indicated by SNR-AXP associations. In this work we discuss the options of the fall-back disk model in view of the recent disk solution employed by Ekşi & Alpar (2003)[12].

2. Fall-back Disk Models

Alpar (2001) [10] proposed a classification of young neutron stars in terms of the absence or

presence and properties of a fall-back disk. According to this model AXPs, SGRs and Dim Thermal Neutron Stars (DTNs) have similar periods because they are in an asymptotic spin-down phase in interaction with a fall-back disk. The different classes represent alternative pathways of neutron stars. Radio pulsars have no disks or encounter very low mass inflow rates while Radio Quiet Neutron Stars (RQNS) have such high mass inflow rates that their pulse periods are obscured by the dense medium around. This model employs constant mass inflow rate corresponding to a constant equilibrium period. As a fall-back disk is not replenished, the mass inflow rate of the disk should decline while the disk diminishes. The mass flow rates characterizing different evolutionary paths given in this model are then representative values and are to be replaced by the initial mass of the disk in a time-dependent model.

Chatterjee et al. (2000) (CHN [9]) employed a time-dependent thin disk model [28,29] in which the angular momentum of the disk is constant and the mass of the disk declines with a power-law. Accordingly, the mass inflow rate also declines as a power-law

$$\dot{M} = \dot{M}_0 \left(\frac{t}{T_0} \right)^{-\alpha} \quad (1)$$

where \dot{M}_0 is the initial inflow rate and T_0 is taken to be the dynamical time scale, $T_d \sim \Omega_K^{-1}$ at the initial inner radius of the disk. The power-law index α depends on the opacity regime. For opacities of the form $\kappa = \kappa_0 \rho^a T^b$, thin disk equations [30] can be solved to obtain

$$\alpha = \frac{18a - 4b + 38}{17a - 2b + 32}. \quad (2)$$

This yields, $\alpha = 19/16$ for electron scattering opacity ($a = b = 0$), and $\alpha = 5/4$ for bound-free opacity ($a = 1$ and $b = -7/2$). CHN assumed electron scattering to be dominating, but employed $\alpha = 7/6$ for analytical convenience. As the mass inflow rate declines, the equilibrium period increases and the system can never reach a true equilibrium. AXPs are those systems accreting from a diminishing disk while spinning down towards an ever receding equilibrium (quasi-equilibrium stage).

Francischelli & Wijers (2002)[31] criticized the CHN model, noting that whether the disk model can produce AXPs or not is sensitively dependent on the value of the power-law index α . They showed that after ~ 20 years, the disk is bound-free opacity dominated and for the corresponding α , the model did not produce AXPs with the nominal disk mass employed by CHN ($M_0 = 0.006 M_\odot$). They also criticized the torque model employed in the CHN model, arguing that it is too efficient and showed that less efficient torque models can not produce AXPs (see also [32]).

While Fracischelli & Wijers (2002) [31] is right in arguing that α is critical in determining the period evolution for a single initial disk mass, it is not possible to reject the fall-back disk model on this ground. For every α , one can find an appropriate initial mass flow rate \dot{M}_0 (corresponding to an initial disk mass M_0) that leads to a period evolution in which the system enters the quasi-equilibrium stage at $P \cong 5$ s, producing an AXP. The time at which the transition to the quasi-equilibrium stage occurs, T_{tr} , and the mass flow rate at this moment, \dot{M}_{tr} , remains nearly the same for each α and its corresponding \dot{M}_0 . One can obtain AXPs with the CHN model by employing $\alpha = 5/4$ corresponding to bound-free opacity as long as one assumes an initial disk mass of $M_0 \cong 0.069 M_\odot$. This larger initial disk mass does not lead to higher luminosity at the accretion stage because the disk decays more rapidly with the corresponding α .

Ekşi & Alpar (2003) (EA03 [12]) employed another time-dependent thin disk solution [28,33] for the evolution of the disk during the propeller stage. In this solution mass of the disk is constant while the angular momentum of the disk increases due to the torque

$$\dot{J} = \dot{J}_0 \left(\frac{t}{T_0} \right)^{-\beta} \quad (3)$$

at the inner boundary of the disk where

$$\beta = \frac{14a + 22}{15a - 2b + 28}. \quad (4)$$

For electron scattering opacity $\beta = 11/14$ and for bound-free opacity dominated disks $\beta = 18/25$.

In this solution mass flows outwards, but Pringle (1991)[33] numerically shows solutions in which the mass flows inwards, and a torque as given in Eq. (3) is necessary to stop accretion at the inner boundary. Pringle (1981) [34] mentioned that “such a solution might represent a disc around a magnetized star which is rotating sufficiently rapidly that its angular velocity exceeds the Keplerian velocity at the magnetosphere” - i.e. a propeller. Assuming the inner radius R_m scales with the Alfvén radius $R_A \propto \dot{M}^{-2/7}$ and referring $\dot{J} \propto \sqrt{GMR_m\dot{M}} \propto \dot{M}^{6/7}$, one can calculate an effective α for the propeller regime. For electron-scattering opacity one obtains $\alpha_p = 11/12$ and for bound-free opacity $\alpha_p = 21/25$. EA03 used another scaling for determining the inner radius in the propeller regime $R_m \propto \dot{M}^{-1/5}$ [35]. This implementation for the inner radius yields $\dot{J} \propto \dot{M}^{9/10}$ and one obtains $\alpha_p = 55/63$ for the electron scattering opacity and $\alpha_p = 4/5$ for the bound-free opacity.

For the sake of consistency with the constant disk mass solution, EA03 assumed that the inflowing matter flung away by the magnetosphere of the neutron star returns back to the disk somewhere away from the inner boundary of the disk. The mass flow rate in the propeller type of solution declines with a softer power-law index than it does in the accretion type of solution because in the former case matter cannot accrete and the decrease in the flow rate is only by the viscous spreading of the disk. Conservation of the mass of the disk during the propeller stage makes it possible to obtain AXPs with a 50 times smaller initial disk masses than required by the CHN model. Again, this does not mean that mass flow rate during the accretion stage will be smaller by this order of magnitude.

The luminosities of AXPs being less than $L_X \simeq 7 \times 10^{35}$ erg s⁻¹ imply that less than $\dot{M}_X \simeq 4 \times 10^{15}$ g s⁻¹ of matter is accreting onto these objects. As Menou et al. (2001) [36] have shown, fall-back disks with $\dot{M} \lesssim 10^{16}$ g s⁻¹ are neutralized and can not be accreting. This means that AXPs must have mass flow rates in excess of $\sim 10^{16}$ g s⁻¹ and only a fraction η of the inflowing matter should accrete onto the neutron star. In CHN model $\dot{M}_{tr} \gtrsim 10^{16}$ g s⁻¹ and one needs

to assume that a fraction $\eta \cong 0.1$ of the inflowing mass accretes onto the neutron star, in order to explain the observed luminosities of $L_X \lesssim 10^{36}$ erg s⁻¹. In EA03 model one needs to assume $\eta \sim 0.01$ in order to explain the luminosities because the system reaches the quasi-equilibrium stage rather rapidly, at an epoch during which the mass flow rate is still high. The difference is due to the different formulations employed for the magnetic radius of the disk during the propeller stage which in turn effects the magnitude of the torque and the time the system reaches the quasi-equilibrium stage.

3. The Model Equations

For finding the inner radius, we follow the EA03 model [12]. The system initially is in the super-critical propeller regime and the inner radius of the disk is determined by

$$\begin{aligned} R_m &= (\mu^2 \kappa_{es} / 8\pi \sqrt{2} c \Omega_*)^{1/6} \\ &= 31 R_* \mu_{30}^{1/3} (P_0 / 15 \text{ms})^{1/6} \end{aligned} \quad (5)$$

while the corotation radius is $R_c \simeq 10 R_* (P / 15 \text{ms})^{2/3}$. The dynamical time scale $T_d \equiv 1 / \Omega_K(R_m)$ correspondingly is $T_d = 1.23 \times 10^{-2} \mu_{30}^{1/6} (P_0 / 15 \text{ms})^{1/4}$ which is one order of magnitude greater than that employed by CHN. Greater T_0 makes it possible to obtain the same mass flow history (and same \dot{M}_{tr}) starting with a smaller \dot{M}_0 by (and smaller M_0). The calculations of Pringle (1991)[33] show that the power-law evolution starts after a viscous time scale rather than the dynamical time-scale employed in early models. Considering the ambiguity in T_0 , one should not attempt to be very definite on the initial mass of the disk in models employing the power-law evolution.

We model the disk torque as

$$\dot{J}_* = \sqrt{GMR_m\dot{M}}(1 - \omega_*/\omega_{eq}) \quad (6)$$

where ω_{eq} is the equilibrium fastness parameter which we take to be 0.85 in this work. As noted by [31] and [32], this is a very efficient propeller torque model compared to some other available propeller torques obtained through simple energy or angular momentum arguments [21,22,23]. Recently Ikhsanov [26] revived the propeller model

of Davies & Pringle (1981) [24] which incorporates $\dot{J}_* \propto -\omega_*$. This model agrees with Eq. (6) in the large ω_* limit. In the form we adopt, the torque entails asymptotic approach to rotational equilibrium which is necessary for addressing the period clustering of AXPs. The recent numerical work [35] also indicate an efficient propeller torque ($\dot{J}_* \propto -\Omega_*^{4/3}$).

Being aware that modifications in α can be compensated by modifying the initial inflow rate which in turn depends on the uncertain value of the dynamical time scale, we simplify the EA03 model as follows:

$$\alpha = \begin{cases} 5/4 & \text{if } \omega_* < 1; \\ 4/5 & \text{if } \omega_* > 1. \end{cases} \quad (7)$$

The value 5/4 is motivated by the accretor type of solution and the value 4/5 is motivated by the propeller type of solution. Both values correspond to the bound free opacity regime. The electron scattering opacity prevails only in the ~ 20 years and does not effect the period evolution significantly.

4. Results and Discussion

We have followed the evolution of the star-disk system for 10^5 years, solving the torque equation (6) numerically with the adaptive step-size Runge-Kutta method [37].

In Figure (1) we show five evolutionary tracks on the $P - \dot{P}$ diagram that can cover seven AXP/SGRs for which period derivatives are available. Figure (2) shows the corresponding period evolutions for each of the tracks in Figure (1), tracks with the same numbers having the same parameters. We assumed the radius and mass of the neutron star to be $R_* = 10^6$ cm and $M_* = 1.4M_\odot$, respectively. Initial period is $P_0 = 15$ ms and we have assumed that the disk neutralizes when the mass flow rate drops below 10^{16} g s $^{-1}$ [36].

Extending the analysis to disk evolution in the propeller regime provides favorable effective power-law indices $\alpha_{\text{eff}} < 1$. However, in the availability of choosing a larger initial disk mass one can always obtain AXPs even if the disk loses mass with $\alpha = 5/4$.

Considering the effect of the radiation pressure on the inner radius indicates a larger dynamical time scale, T_0 . With a larger T_0 , one can produce the same mass flow history with a smaller initial mass inflow rate corresponding to a smaller initial disk mass. The numerical calculations (see [33]) indicate that the power-law evolution corresponding to the self-similar solutions start in a viscous time-scale which is still larger than the dynamical time scale. Given the ambiguity in T_0 , estimations of initial disk mass from power-law models discussed above, should not be taken very seriously.

Modifications in α and T_0 can always be compensated in the sense that one can obtain the same period evolution by tuning the initial mass flow rate. Such modifications then lead to the same mass flow rate at the quasi-equilibrium stage at which the system is observed as an AXP.

The part of the model that is critical in determining whether fall-back disks can produce AXPs or not is the propeller torque as first shown by [31]. If the torque model is inefficient, one must then employ a high mass flow history to spin-down the neutron star to AXP periods which in turn results with very high luminosities in the quasi-equilibrium stage. As the torque is dependent on the inner radius of the disk, the results are effected by the uncertain factors ($\xi = R_m/R_A$) in determining the inner radius.

It is possible that a fall-back disk loses mass also in the propeller regime because of the propeller effect itself, depending on the fastness parameter. The simple power-law models for the flow rate are working models which do not take into account the coupling of the star to the structure of the disk. A more realistic treatment of fall-back disks would consider the effects of the magnetic field on disk evolution, realistic opacities, including the iron line opacity [38] for the fall-back disk which is rich in heavy elements, and the dependence of the accretion rate itself on the motion of the inner radius $\dot{M} = 2\pi R_m \Sigma(R_m) [\dot{R}_m - v_r(R_m)]$ (see [39]). All of these effects would result in time evolution of the disk that cannot be described by simple power laws. Taking account of the complexities of the real problem is not likely to produce qualitative disagreement with

the evolutionary timescales as long as the torque is efficient.

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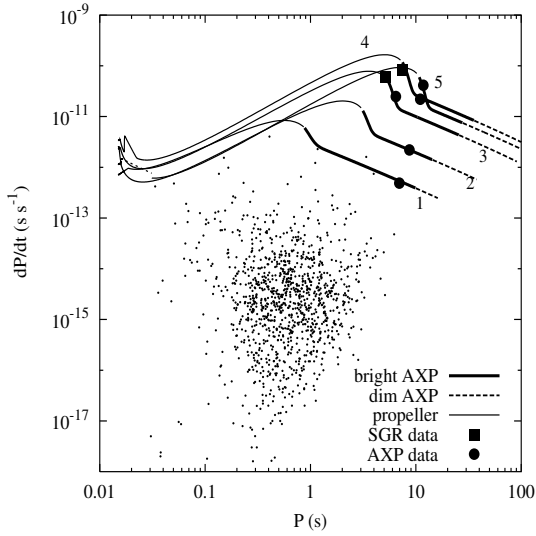


Figure 1. Evolutionary tracks in the $P - \dot{P}$ diagram covering all AXPs and two of the SGRs with known period derivatives. The *thin solid lines* show the rapid spindown phase, the *thick solid lines* stand for the “bright AXP” stage and the *dashed lines* show the “dim AXP” stage after the disk is neutralized ($\dot{M} < 10^{16} \text{ g s}^{-1}$). The *dots* denote the radio pulsars. AXPs are shown by *filled circles* and SGRs by *squares*. With different μ_{30} and \dot{M}_0 , the model can cover all AXPs and SGRs. The parameters for each track are as follows: (1) $B_{12} = 1.8$, $\dot{M}_0 = 5 \times 10^{28} \text{ g s}^{-1}$; (2) $B_{12} = 2.8$, $\dot{M}_0 = 1 \times 10^{28} \text{ g s}^{-1}$; (3) $B_{12} = 5.5$, $\dot{M}_0 = 5 \times 10^{27} \text{ g s}^{-1}$; (4) $B_{12} = 8$, $\dot{M}_0 = 3.2 \times 10^{27} \text{ g s}^{-1}$; (5) $B_{12} = 6$, $\dot{M}_0 = 2 \times 10^{27} \text{ g s}^{-1}$. The initial period is 15 ms for all simulations.

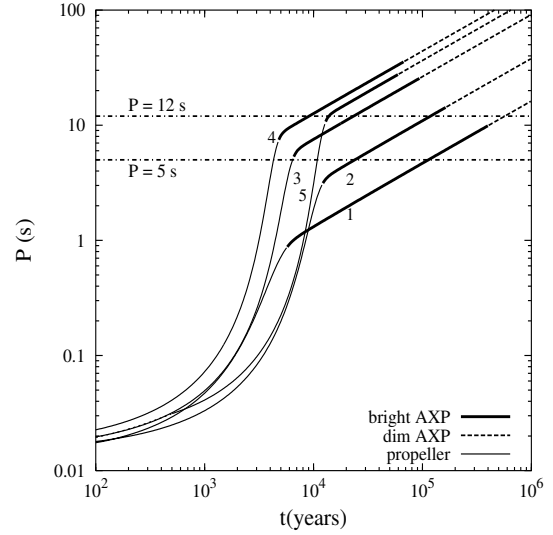


Figure 2. Period evolution of AXPs. The *thin solid lines* show the rapid spindown phase, the *thick solid lines* stand for the “bright AXP” stage and *dashed lines* show the “dim AXP” stage after the disk is neutralized. The *dashed and dotted horizontal lines* show the period range of AXPs and SGRs. The parameters are the same as in Figure (1) and the numbers labelling the tracks correspond to the numbers of the tracks in Figure (1).